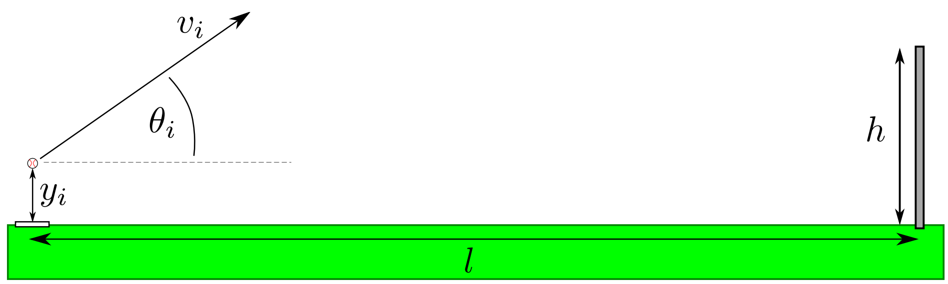
Mechanics - Air Drag - Home run in Colorado vs. Houston

March 25, 2012

The initial conditions for a ideally hit baseball are given as y_i, v_i, and θ_i.



For the ball to clear the fence of height h at a distance l away the air density must be low enough. The force of air drag on the ball is given as,

$$F_{\mathrm{d}} = \frac{1}{2}\rho a d v^2$$

where ρ is the air density, a is the baseball's cross-sectional area, and d is the drag coefficient for a baseball. Calculate the range difference for these two places given $y_i = 1$, $\theta_i = 38^0$,$v_i = 40\frac{\mathrm{m}}{\mathrm{s}}$. Plot their x and y trajectories on the same plot. Then find an h and an l that would allow one to be a home-run and the other to bounce back off the fence into the field.

Solution:

The Colorado/Houston air densities are 1.004357654 $\frac{\mathrm{kg}}{\mathrm{m}^3}$ and 1.223810602 $\frac{\mathrm{kg}}{\mathrm{m}^3}$ respectively.

The cross-sectional area of a baseball is 0.00456036731 m^2.

The drag coefficient for a baseball is .3.

The mass of the ball is $m = .145$.

The equations of motion for the ball are

$$m\ddot{x} = -\frac{1}{2}\rho a d\dot{x}^2$$

$$m\ddot{y} = -\frac{1}{2}\rho a d\dot{y}^2 - mg$$

Continue in the x direction, $\dot{x} \to v_x$

$$m\frac{dv_x}{dt} = -\frac{1}{2}\rho a d\ddot{x}^2$$

$$m\int\frac{1}{v_x^2}dv_x=-\frac{1}{2}\rho ad\int dt$$

$$m\left(-\frac{1}{v_x}+C_1\right)=-\frac{1}{2}\rho adt$$

but C_1 is just $\frac{1}{v_{x0}}$

$$m\left(\frac{1}{v_{x0}}-\frac{1}{v_x}\right)=-\frac{1}{2}\rho adt$$

$$v_x\left(t\right)=\left(\frac{1}{v_{x0}}+\frac{1}{2m}\rho adt\right)^{-1}$$

$$\frac{dx}{dt}=\left(\frac{1}{v_{x0}}+\frac{1}{2m}\rho adt\right)^{-1}$$

$$\int dx=\int\left(\frac{1}{v_{x0}}+\frac{\rho ad}{2m}t\right)^{-1}dt$$

$$x\left(t\right)=\frac{2m}{\rho da}\ln\left(1+\frac{\rho dav_{x0}}{2m}t\right)$$

Continue in the y direction, $\dot{y}\to v_y$

$$m\ddot{y}=-\frac{1}{2}\rho ad\dot{y}^2-mg$$

$$m\frac{dv_y}{dt}=-\frac{1}{2}\rho adv_y^2-mg$$

$$\int\frac{1}{-\frac{\rho ad}{2m}v_y^2-g}dv_y=\int dt$$

$$\arctan\left(\frac{\sqrt{\frac{\rho ad}{2m}}v_{y0}}{\sqrt{g}}\right)-\arctan\left(\frac{\sqrt{\frac{\rho ad}{2m}}v_y\left(t\right)}{\sqrt{g}}\right)=\sqrt{\frac{\rho adg}{2m}}t$$

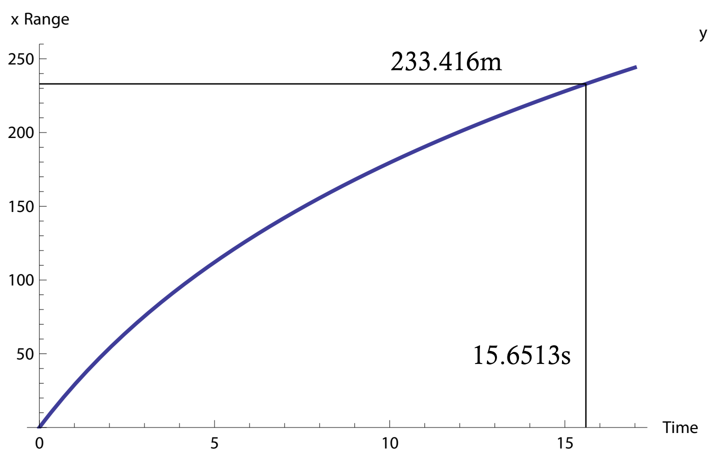
$$\arctan\left(\frac{\sqrt{\frac{\rho ad}{2m}}v_y\left(t\right)}{\sqrt{g}}\right)=\arctan\left(\frac{\sqrt{\frac{\rho ad}{2m}}v_{y0}}{\sqrt{g}}\right)-\sqrt{\frac{\rho adg}{2m}}t$$

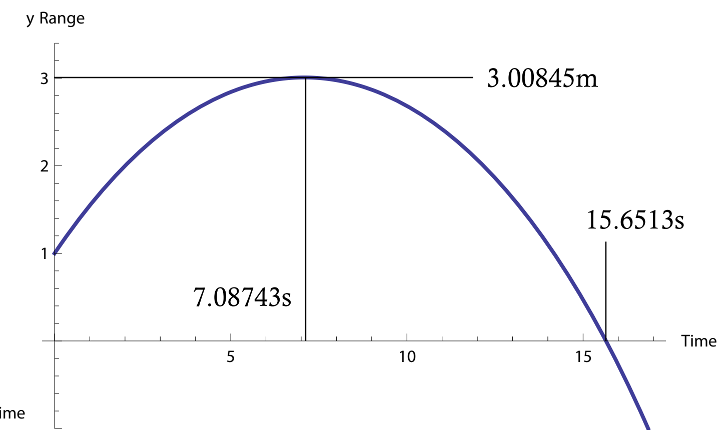
$$v_y\left(t\right)=\sqrt{\frac{2mg}{\rho ad}}\tan\left[\arctan\left(\frac{\sqrt{\frac{\rho ad}{2m}}v_{y0}}{\sqrt{g}}\right)-\sqrt{\frac{\rho adg}{2m}}t\right]$$

$$y\left(t\right)=\sqrt{\frac{2mg}{\rho ad}}\int\tan\left[\arctan\left(\sqrt{\frac{\rho ad}{2mg}}v_{y0}\right)-\sqrt{\frac{\rho adg}{2m}}t\right]dt$$

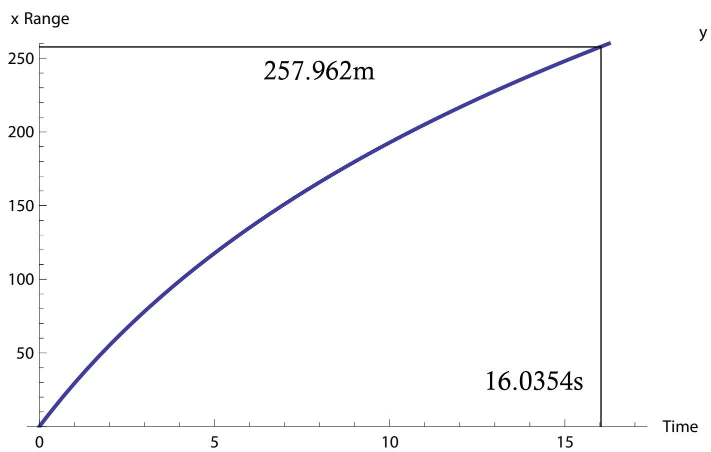
$$y\left(t\right)=\sqrt{\frac{m^2g}{\rho^2a^2d^2}}\left\{\ln\left(1+\frac{ad\rho v_{y0}^2}{2gm}\right)+2\ln\left[\frac{\sqrt{2}\cos\left(\sqrt{\frac{ad\rho}{2m}}t\right)+\sqrt{\frac{adr}{gm}}v_{y0}\sin\left(\sqrt{\frac{ad\rho}{2m}}t\right)}{\sqrt{2+\frac{ad\rho v_{y0}^2}{gm}}}\right]\right\}+y_0$$

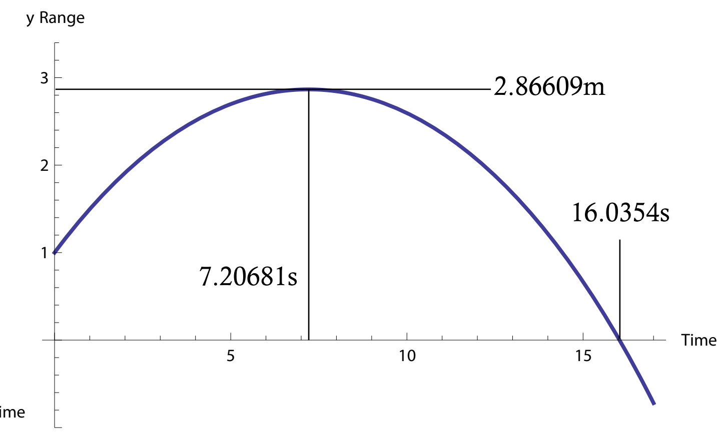
So in Houston we get a time of flight of 15.6513s, max height 3.00845m, time of peak 7.08743s, and x range of 233.416m.



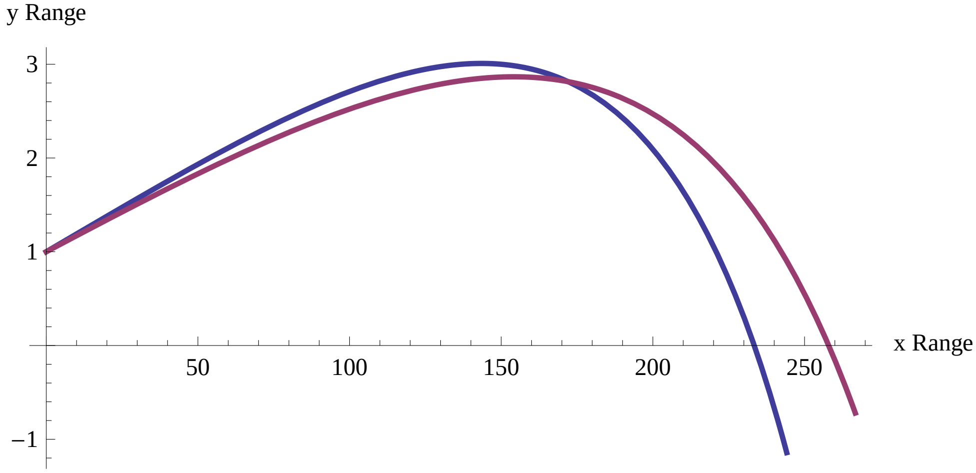


So in Colorado we get a time of flight of 16.0354s, max height 2.86609m, time of peak 7.20681s, and x range of 257.962m.





Their x and y trajectories are



An h and l that would separate these two trajectories is $h \simeq 2.28\text{m}$ and $l = 200\text{m}$